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Network structure and the diffusion of knowledge

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Abstract

This paper models knowledge diffusion as a barter process in which agents exchange different types of knowledge. This is intended to capture the observed practice of informal knowledge trading. Agents are located on a network and are directly connected with a small number of other agents. Agents repeatedly meet those with whom direct connections exist and trade if mutually profitable trades exist. In this way knowledge diffuses throughout the economy. We examine the relationship between network architecture and diffusion performance. We consider the space of structures that fall between, at one extreme, a network in which every agent is connected to n nearest neighbours, and at the other extreme a network with each agent being connected to, on average, n randomly chosen agents. We find that the performance of the system exhibits clear ‘small world’ properties, in that the steady-state level of average knowledge is maximal when the structure is a small world (that is, when most connections are local, but roughly 10 percent of them are long distance). The variance of knowledge levels among agents is maximal in the small world region, whereas the coefficient of variation is minimal. We explain these results as reflecting the dynamics of knowledge transmission as affected by the architecture of connections among agents.

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1. Introduction

One of the remarkable features of a market is its ability to process information. Even with the myriad factors that can influence economic phenomena, in a well-functioning

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market economy agents need only pay attention to one piece of information for each good, namely its price. While economists understand that, and in an abstract sense how, a market processes information, there has been little explicit examination of it. In particular, network or communication structures under which agents operate and transmit or exchange knowledge and information have received little attention. But the details of who is connected to whom will clearly affect what type of information is passed, how much, and how efficiently. All of these can have an effect on the aggregate performance of the system being modelled. Two questions immediately arise: first, if the network structure is exogenous, how do the structural properties of the network affect aggregate outcomes? and second, if network formation is endogenous, what structures are likely to emerge? The current paper addresses the first of these questions, namely how the communication network structure affects the aggregate performance of the system.¹

We take as the motivating application a system of knowledge diffusion. In our model knowledge diffuses through barter exchange among pairs of agents, and aggregate performance is measured as the mean knowledge level over all agents. The parameter we vary is the degree of spatial randomness in the network through which knowledge circulates. We consider the space of structures that fall between, at one extreme, a network in which every agent is connected to n nearest neighbours, and at the other extreme a network with each agent being connected to, on average, n randomly chosen agents. In terms of the aggregate performance of the system, we find that one region of the space stands out. This region corresponds to the ‘small world’ network structure as defined formally by Watts and Strogatz (1998), and informally by Milgram (1967).

While the literature on local interaction uses different structures to model communication networks, there are two dominant designs. In generalized Ising models all interactions are spatially local, and the interaction structure is completely regular. Every agent is directly connected to the same small number of his nearest neighbours.² At the other extreme are models based on random graph theory. Here, in principle any agent could be connected to any other, and there is no spatial structure imposed on the connections. In this sense interactions can be seen as spatially global.³ Typically local interaction models do not address issues of network ‘topology’ or do so by

¹ See for example Jackson and Wolinsky (1996), Jackson and Dutta (2001), Bala and Goyal (2000), Goyal and Moraga-González (2001), Weisbuch et al. (2000) on the second question.

² This structure has been used to examine macro-economic dynamics (Durlauf, 1991, 1993); technology diffusion (Allen, 1982; An and Kiefer, 1993; David et al., 1998); the effectiveness of prices in stabilizing an economy (Föllmer, 1974); criminal behaviour (Glaeser et al., 1996); equilibrium selection in Coordination Games (Anderlini and Ianni, 1996). Clustering in behaviour is a common result: in steady states we observe neighbourhoods in which agents’ behaviour is similar to that of nearby agents, but different from that of agents in other neighbourhoods. A recent survey can be found in Kirman (1998).

³ This general structure of random global or non-ordered connections has been used to examine, for example, coalition formation (Kirman, 1983; Ioannides, 1990).

addressing only the issue of network density.⁴ The focus in this paper, by contrast, is specifically on the architecture of the connections, and we deliberately hold the number of connections constant throughout the analysis.

The exchange of knowledge is at the centre of the model we develop below. Recent studies of knowledge and its transfer among agents emphasize the importance of tacit knowledge and the crucial role of face-to-face interactions. If this knowledge is diffused therefore, models of its diffusion must take explicit account of the structure of connections between agents.⁵ While not denying the importance of other kinds of knowledge and modes of diffusion, in this paper we focus on knowledge that is exchanged in this way. In-depth empirical research indicates that most industries have a well-established informal network through which knowledge is traded: even among competitors knowledge is exchanged, but in a barter arrangement (von Hippel, 1987; Hicks, 1995; Schrader, 1991). For information given up, through conference presentations, workshops or conversations in a bar, there is a quid pro quo—information is expected in return. It is this kind of knowledge trading and the network structure in which it takes place that we model in the next section.

2. The model

The model we propose explores how the topology of agents' interactions influences aggregate regularities. We are not concerned here with the diffusion of radical innovations, and particularly not product innovations. The model captures effects of incremental innovations, and their diffusion among a network of heterogeneous agents. Agents are characterized by knowledge endowments which evolve over time through a simple process of barter exchange. We treat knowledge as a vector of knowledge types, unlike many models in which knowledge is a simple scalar, and which therefore cannot capture knowledge trading behaviour.

2.1. The network

Let $I = \{1, \dots, N\}$ denote a finite set of agents. For any $i, j \in I$, define the binary variable $\chi(i, j)$ to take the value $\chi(i, j) = 1$ if a connection exists between i and j ,

⁴ There are exceptions. Midgley et al. (1992) have a model with spatial characteristics—agents are located in cliques, within an industry, or outside the industry. They find that increasing the number of connections in the model increases diffusion speeds, though different types of connections (intra-clique; inter-clique; outside the industry) have different effects. This is addressed in a much more systematic way in Bala and Goyal (1998, 2001) who study the related problem of learning the payoff associated with different actions in a structured society. If a connected society is divided into sub-groups with more links between agents within the same group than across different groups, different behaviours can coexist, whereas uniformity appears in a global interaction context. They also provide structural conditions under which all individuals of the same type eventually choose the same behaviour (see also Goyal and Janssen, 1996). The issue has also arisen in game theory models (Young, 1998; Chwe, 1999, 2000; Morris, 2000).

⁵ See Cowan and Foray (1997). Empirical work supports the contention that significant amounts and types of knowledge travel only short distances, which suggests that diffusion involves face-to-face communication. See for example Feldman (1994), Prevezer and Swann (1996) or Jaffe et al. (1993).

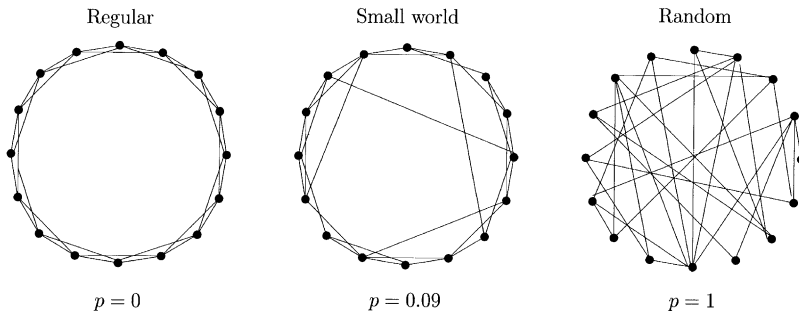


Fig. 1. The transition from a locally ordered structure to a disordered one via a small world.

and $\chi(i, j) = 0$, otherwise. The network $G = \{\chi(i, j); i, j \in I\}$ is the list of all pairwise relationships between agents. The neighbourhood of i is the set $\Gamma_i = \{j \in I : \chi(i, j) = 1\}$. A path in G connecting i and j is a set of pairwise relationships $\{(i, i_1), \dots, (i_k, j)\}$ such that $\chi(i, i_1) = \dots = \chi(i_k, j) = 1$. Finally define the distance $d(i, j)$ between i and j to be the length of the shortest path between them.

The following algorithm (Watts and Strogatz, 1998) allows us to construct a family of constant density graphs which lie between, at one extreme, a nearest-neighbour graph on a periodic lattice, and at the other extreme a random graph with uniform degree.

Create the regular periodic lattice with n nearest neighbours (n even). Sequentially, consider each edge of the graph; with probability p disconnect one of its vertices, and connect it to a vertex chosen uniformly at random. Check both that vertices do not get self-connected, and that no two vertices are connected more than once. For large graphs, this procedure ensures that G is connected. By tuning p , we vary the graph structure from completely regular ($p = 0$), through intermediate states, ($0 < p < 1$), to completely disordered ($p = 1$). This creates variation in the number of edges per agent, but maintains an average of n connections per agent and a total of $Nn/2$ edges, $\forall p$. Define $G(n, p)$ to be the graph produced by this algorithm. Fig. 1 depicts three illustrative configurations with increasing disorder as p is increased, for $N = 16$ and $n = 4$.

Watts and Strogatz (1998) point out that the structural properties of $G(n, p)$ -graphs are intuitively captured by the concepts of average path length and average cliquishness. Define the cliquishness of a set $S \subseteq I$ to be the proportion of pairwise relationships in S over the total possible number of relationships, that is

$$c\ell(S) = \frac{\sum_{i,j \in S} \chi(i, j)}{\#S(\#S - 1)/2}. \tag{1}$$

In a social network, it is the share of friends of one individual who are also friends of each other. Cliquishness can be used to measure local coherence or redundancy by taking S to be the neighbourhood of an agent. Then the local coherence in the network is measured by the average neighbourhood cliquishness $C(p) = \sum_{i \in I} c\ell(\Gamma_i)/N$. Average path length is $L(p) = \sum_{i,j \in I} d(i, j)/(N(N - 1)/2)$, the average number of steps separating two randomly chosen agents. Though a natural conjecture is that cliquishness

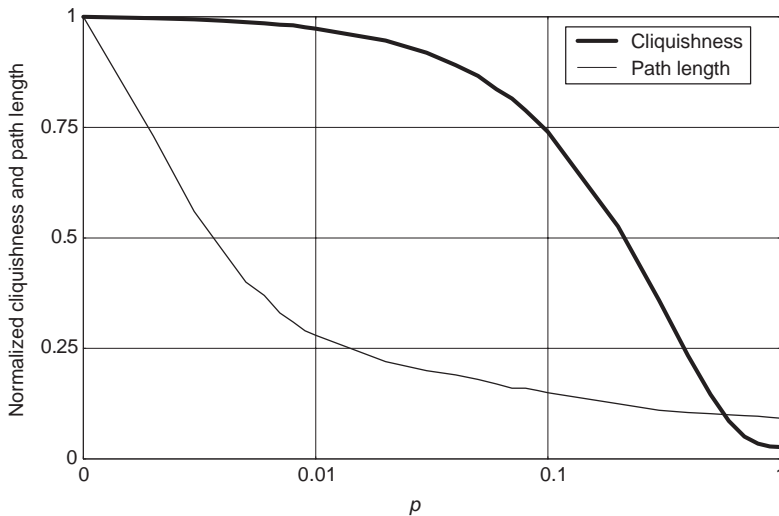


Fig. 2. Average cliquishness and average path length as function of p .

and path length are strongly correlated, there is a non-negligible interval for p over which $L(p) \simeq L(1)$ yet $C(p) \gg C(1)$. This interval constitutes the small-world region. It arises because when the number of long distance links is small, their marginal effect on average path length is large: introducing a long-range edge provides a shortcut not only between the two vertices that this link connects, but also for their immediate neighbours, the neighbours of those neighbours and so on. By contrast removing one local link affects the cliquishness of only a small number of neighbourhoods and therefore has little effect on the population average.

The evolution of path length and clique size with p is shown in Fig. 2, for a graph of $N = 500$ vertices, each vertex having on average $n = 10$ nearest neighbours. For the sake of clarity, we plot the averaged normalized values $L(p)/L(0)$ and $C(p)/C(0)$ over a sample of 500 different graphs. Normalized average cliquishness remains almost constant when p is reasonably small and falls slowly for large values of p . By contrast, average path length falls quickly for very small p values and then flattens out. Hence, for $p \in [0.01, 0.1]$, cliquishness and path length diverge, creating a small world region in the space of network structures.

2.2. Knowledge barter

We assume that knowledge transfer takes place through a myopic barter exchange. In this model when two agents meet, the decision is simply whether to trade or to walk away. For each agent this decision is based only on whether or not the other agent has something to offer, and not on the amount he has to offer. This places minimal demands on an agent's ability to assess a priori the knowledge level of a potential partner, and admits the important property of knowledge that before any knowledge

exchange there is uncertainty about exactly what will be gained. Thus it is in principle possible for an agent to give a lot, but receive only a little in return.

Agent $i \in I$ is characterized by a knowledge vector, $v_i = (v_{i,c})$ with $c = 1, \dots, \ell$, that evolves over time through trade. Agents i and j interact if and only if there is a direct connection between them and trading is mutually advantageous. Assume $j \in \Gamma_i$ and let $n(i, j) = \#\{c : v_{i,c} > v_{j,c}\}$ be the number of knowledge categories in which i strictly dominates j . Agent j is only interested in trading with i if $n(i, j) > 0$, and the symmetric condition is true for i . Trade therefore takes place if and only if $j \in \Gamma_i$ and

$$\min\{n(i, j), n(j, i)\} > 0. \quad (2)$$

Condition (2) states that there must be a double coincidence of wants for barter to take place. When this holds agents barter as many categories as possible. If (2) holds but $n(i, j) \neq n(j, i)$, it is assumed that the categories involved in trading are randomly chosen, with uniform probability. Each agent therefore gives and receives some knowledge in a number of categories equal to $\min\{n(i, j), n(j, i)\}$. In keeping with much recent work on the economics of knowledge, we have assumed that knowledge is only partly assimilable, hence trading, when it occurs, results in a gain for the receiver equal to a constant share $\alpha < 1$ of the knowledge differential.⁶ The non-rivalry property of knowledge implies that there is no loss to the sender. Suppose agent j dominates agent i in category c_1 and i dominates j in c_2 . After the exchange knowledge levels have become

$$\begin{aligned} v_{i,c_1}(t+1) &= v_{i,c_1}(t) + \alpha[v_{j,c_1}(t) - v_{i,c_1}(t)], \\ v_{j,c_1}(t+1) &= v_{j,c_1}(t), \\ v_{j,c_2}(t+1) &= v_{j,c_2}(t) + \alpha[v_{i,c_2}(t) - v_{j,c_2}(t)], \\ v_{i,c_2}(t+1) &= v_{i,c_2}(t). \end{aligned} \quad (3)$$

This trading process continues until all trading possibilities are exhausted, that is, there are no further double coincidences of wants: $\forall i \in I, \forall j \in \Gamma_i : \min\{n(i, j), n(j, i)\} = 0$. This describes the steady state of the process.

It is worth pointing out the implicit assumption that agent utility increases with agent knowledge levels. In a world of rapid technological change, innovation is vital for survival, so this simple assumption captures (or perhaps motivates or explains) many aspects of firm behaviour (see also footnote 7). One might think that the perfectly rational, forward-looking, agent would simply give away all his knowledge to anyone he met, since his own knowledge level, and thus utility, is not thereby decreased, and raising his neighbours' knowledge levels may raise his own in the future. But note that if there are any secondary, competitive effects wherein for example superior knowledge may give a firm some market power (such effects are not explicitly modelled here) then free riding may be an optimal response when others are giving away their knowledge,

⁶ The strictly partial assimilation arises from the fact that absorptive capacity is never perfect (Cohen and Levinthal, 1989, 1990). Because $\alpha < 1$ the model also has the property that knowledge degrades as it is transmitted: the further a piece of information travels, the less value it is to the recipient.

which suggests that an economy of knowledge gift-giving is not supportable.⁷ A more telling argument against modelling knowledge transfer as a gift, however, is simply that knowledge agents *do not* give away their knowledge. Empirical studies do find that there is a quid pro quo, and that those who never have knowledge to offer are excluded from the settings in which these exchanges take place.⁸

3. Numerical analysis

This section describes the design and results of the numerical implementation of the model.

3.1. Settings

We consider a population of $N=500$ agents with $n=10$ links per agent. At the outset the agents are placed on a graph created according to the algorithm described above. The graph is unchanged within a history. Each agent has a 5-category knowledge vector, initialized by setting $v_{i,c}(0) \sim U[0, 1]$. Parameter α , measuring absorptive capacity, is set to 0.1 for every agent. While it is difficult to say in general what would be a reasonable value for α , it turns out (see Section 3.5) that α only influences the behaviour of the model in a minor way. In this model of knowledge diffusion, if all agents have very similar knowledge levels (independently identically distributed in $[0, 1]$) there will be little to diffuse and the performance of the model will be difficult to observe. Thus we introduce 25 highly knowledgeable agents referred to as ‘experts’, each having an initial knowledge value of 10 in only one category.

Each period an agent, $i \in I$ and one of his neighbours $j \in \Gamma_i$ are chosen uniformly at random. All possible trades between these two agents take place.⁹ This process continues until all possible trades have been made, which holds when for all possible pairs, one agent weakly dominates the other in all categories. To implement this weak domination, since with real numbers knowledge levels never become identical, we consider them identical if they differ by less than one percent:

$$0.99 < \frac{v_{i,c}}{v_{j,c}} < 1.01. \quad (4)$$

⁷ If competitive effects exist, then a barter exchange is an attempt by the two firms involved to increase their knowledge levels vis-à-vis the rest of the population of firms, and thus to increase, in the short run at least, their profits. (By contrast a gift would increase the recipient’s profits without increasing the giver’s.) To model this explicitly would add significant complication to the model without adding further insight regarding the effects of different network structures on the diffusion and growth of knowledge. For a game-theoretic model in which this aspect is central, and in which trading knowledge with a circumscribed number of other agents is supported as a supgame equilibrium, see Eaton and Eswaran (1997).

⁸ See for example von Hippel (1987), Schrader (1991), Hicks (1995), Rosenberg (1990), Rosenberg and Nelson (1994), or Pake (1986).

⁹ Not every meeting will result in trades—one member of the pair may dominate the other in all knowledge categories.

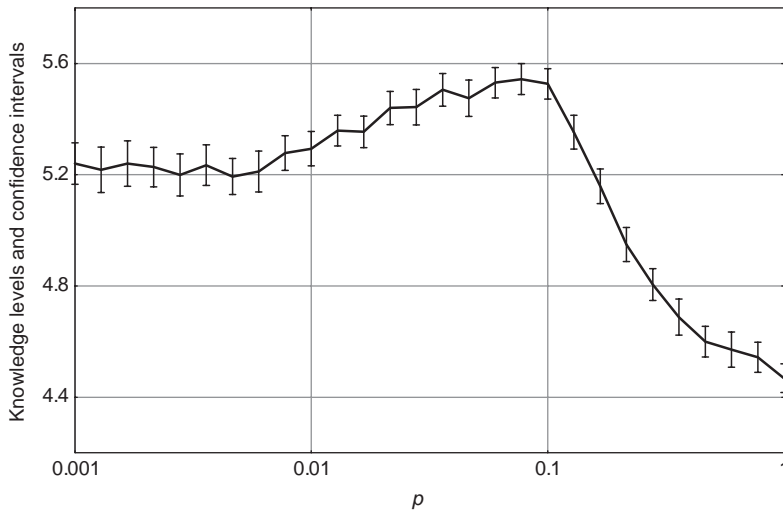


Fig. 3. The economy-wide steady-state average knowledge level μ as a function of the rewiring probability p .

This condition guarantees that the process eventually stops. To examine the space of graphs, we vary the rewiring probability p from 0 to 1. For each p value, 100 different graphs are created and on each graph a single history is run.

We are interested in evaluating the efficiency of the system in distributing knowledge. Agent i 's average knowledge level is $\bar{v}_i(t) = \sum_c v_{i,c}(t)/\ell$. The average level of knowledge in the economy at time t is $\mu(t) = \sum_{i \in I} \bar{v}_i(t)/N$ and the variance in knowledge allocation is $\sigma^2(t) = \sum_{i \in I} \bar{v}_i^2(t)/N - \mu^2(t)$. At the steady state we remove the time index. The curves provided are averages over 100 replications and bars indicate the 95% confidence interval for the true population statistics.

3.2. Small worlds and knowledge levels

Fig. 3 shows the relationship between p and the steady-state performance of the economy μ . The long-run average level of knowledge μ is a non-monotonic function of p , with a clear peak in the small world region. As measured by average knowledge levels, this system performs best at $p = 0.09$.

This result might look counter intuitive, as short path lengths are typically associated with rapid diffusion, and efficient aggregate knowledge growth. But recall that as density is constant, decreasing path length must imply decreasing cliquishness, and what Fig. 3 shows is that there is a tension between the two. A clear implication of this result is that the average path length of a network of relationships is actually not an unequivocal measure of the performance of this structure. Diminishing the distance between members of an organization or economic system by reallocating links does not always improve performance: the architecture of links matters or, put another way, there is value to cliquishness.

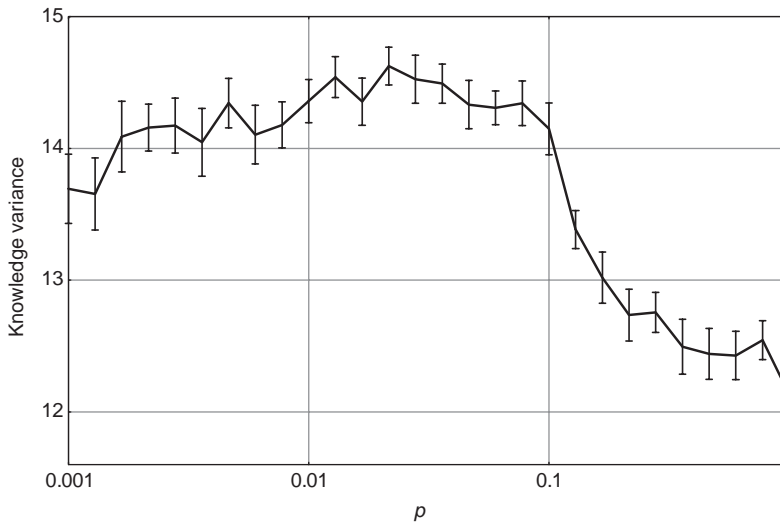


Fig. 4. Agent heterogeneity as measured by the steady-state knowledge variance σ^2 .

3.3. Small worlds and knowledge distribution

We use two measures of the equity in knowledge among agents: variance and coefficient of variation. Fig. 4 shows the relationship between the steady-state variance σ^2 of agents' knowledge and the rewiring probability p .

Heterogeneity and efficiency behave in a similar manner. Again a single-peaked curve obtains; the small world region generates the highest level of disparity in knowledge levels. This suggests a possible trade-off between efficiency and equity—where long-run knowledge levels are highest, knowledge dispersion is highest as well. If there is a strong connection between knowledge and wealth, this may indicate a policy tension.

Variance is a natural measure of dispersion, but if the mean increases simply through some scaling effects, it may be a misleading measure of inequality. In Fig. 5 we show the relationship between network architecture and the equilibrium coefficient of variation of knowledge levels σ/μ .

This measure tells quite a different story. Normalized dispersion of knowledge levels falls in the small world region, suggesting that what is lost in terms of increased heterogeneity as measured by the variance, is more than made-up by an increase in overall knowledge levels. If this is an appropriate measure of dispersion, the policy tension between efficiency and equity dissolves—both are optimized with a small world network.

3.4. Small worlds and diffusion dynamics

To this point our analysis has focused on long-run properties. We now turn to the transitory properties of the model. The speed at which knowledge diffuses is a major

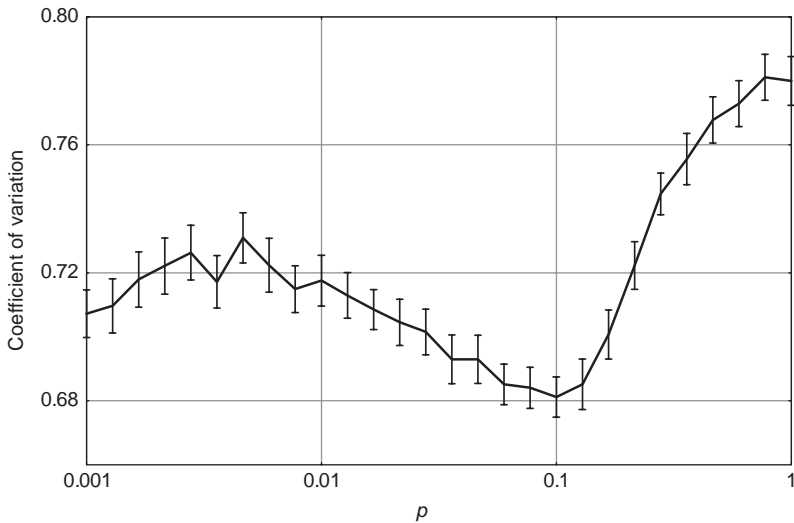


Fig. 5. Agent heterogeneity as measured by the steady-state coefficient of variation of knowledge levels σ/μ .

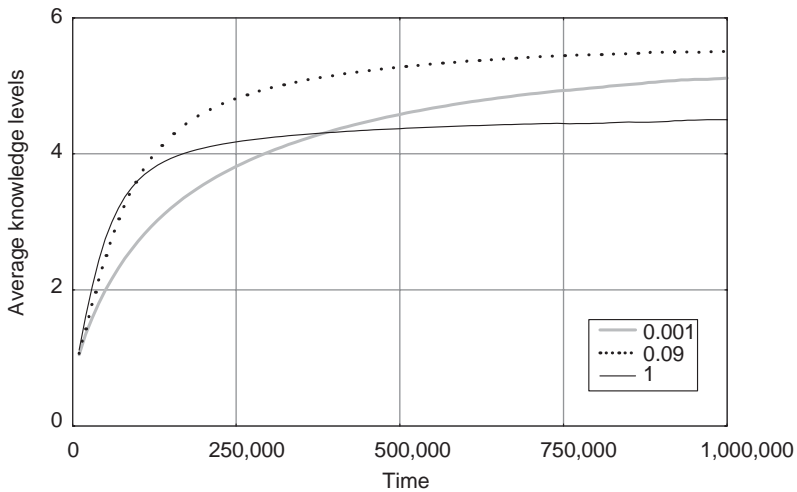


Fig. 6. Diffusion time paths for three characteristic p -values.

policy concern and we can use the model to examine how this is affected by network architecture. Fig. 6 shows the time series of the average knowledge level $\mu(t)$ for three representative values of the rewiring probability p ($p = 0, 0.09$ and 1). Results are reported up to 1,000,000 trading rounds.

When $p = 1$, the network has little local structure but short paths between agents. In early periods of the history, diffusion is rapid. But the process is exhausted at relatively

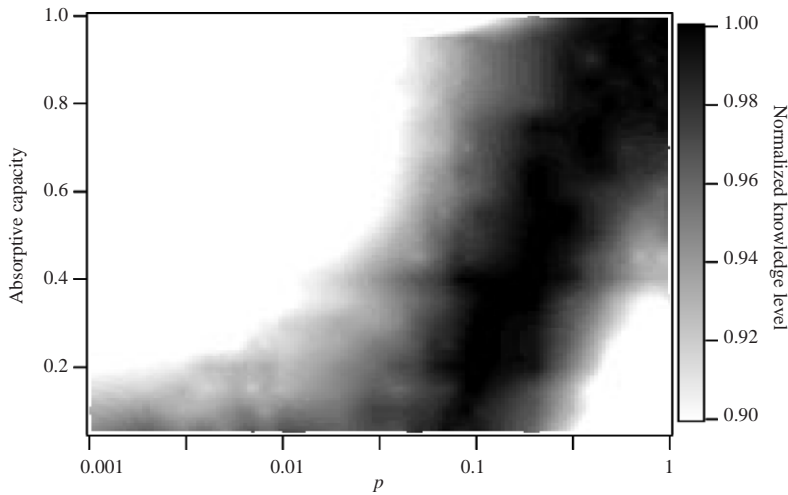


Fig. 7. Average normalized knowledge levels as functions of p and α .

low levels of aggregate knowledge. When $p = 0.001$, the network is locally structured but path lengths are long. Early diffusion is slower than in the random world, but the process continues longer, and reaches a higher aggregate level. Small world networks ($p = 0.09$) have advantages of both: because they have relatively short path lengths diffusion in the early periods is relatively fast; because they are locally structured, trade continues longer than it does in random worlds. Depending on the time horizon of policy measures, different structures will be favoured.

3.5. Sensitivity analysis

In this section we discuss the sensitivity of the results to changes in absorptive capacity and population size, and comment briefly on the role of experts.

Fig. 7 displays the relationship between knowledge levels and p and α as a filled contour plot. This should be read like a map in an atlas: the dark regions show peaks, and the light regions show valleys. Because the values have different magnitudes, we plot normalized results, that is, $\mu(\alpha, p)/\max_p \mu(\alpha, p)$ to show the knowledge maximizing p -value for each α . As α increases from $\alpha = 0.05$ to 1, the ‘optimal’ p moves to the right. However, even for (unrealistically) high absorptive capacities knowledge diffusion is most effective in worlds in which there is still a reasonable amount of cliquishness. The optimal p increases with α because the relative importance of path length and cliquishness changes as α changes. This can be seen by considering the extreme case of $\alpha = 1$. In this case if i trades with j , the two become identical. Agent k therefore, is indifferent between trading with i and trading with j . Thus if k has a link to j , there is no value to k to have a link to i . This says that there is no value to cliquishness— k would be better to have a link to someone that j is not connected to.

Table 1

Aggregate statistics over all network structures and all replications; time to convergence expressed in thousands of periods

Absorptive capacity α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Time to convergence	1450	520	320	265	220	155	120	95	65
Steady-state knowledge levels	3.90	4.65	5.67	6.38	6.69	6.92	7.57	8.08	8.43

So, as α increases the value of cliquishness falls, and thus the relative value of short paths increases.

Parameter α has two additional effects. It changes the overall efficiency of the barter exchange process, and it affects the speed of convergence. Regarding the first effect, the theoretical upper bound of the long-run knowledge level is 10, whereas the levels achieved in this economy are in the neighbourhood of 5 (see Fig. 3). Thus significant trading opportunities are not being captured, which is a well-known shortcoming of barter economies. Even so as α increases, each trade transfers more knowledge so increasing α increases the final knowledge levels. Regarding the second effect, as α increases, any trade brings agents closer together in their knowledge levels. This implies that trading opportunities are exhausted more quickly, and the economy converges faster. In Table 1 we show the effects of α on both speed of convergence and the aggregate long-run knowledge levels.

The role of experts is important as they are ultimately the source of the innovations or knowledge that is being diffused. Consequently, they form local peaks in the knowledge distribution, and agents directly connected to them benefit, becoming second-order experts. In a cliquish graph the result is cliques surrounding the experts the members of which all have high knowledge levels. Thus cliquishness has its biggest effects when an expert is included in the clique. In a world in which there are no experts then in aggregate the value of cliquishness is much attenuated and we would expect that the effects of path length dominate. This raises the interesting question of how much asymmetry there must be in the initial knowledge levels before the effects of cliquishness are felt at the global level. This question is beyond the scope of this paper.¹⁰

Finally we note that the results are more general than the particular family of graphs we consider. First, the size of the population is not important. What is important is that the graph is sparse, so that any agent is directly connected to only a very small proportion of the population (in the range of 2%).¹¹ Regardless of the size of the population or the algorithm used to generate the graphs, the arguments regarding the

¹⁰ The structure we use, based on explicit experts, is actually an extreme case of a skewed distribution of initial knowledge levels. Initial exploration shows that the results hold for an exponential distribution $\Pr\{v_{i,k} \leq x\} = 1 - \exp(-\lambda x)$ of initial knowledge levels for a wide parameter range, $\lambda \in [0.2, 5]$.

¹¹ To generate results from the sort of experiment we do here, agents must have enough neighbours that we can see variations in cliquishness as p is varied. This implies an effective lower bound on the simulated population size of about 250.

relative value of cliquishness and path length will apply to any family which is sparse and which has constant density.¹²

3.6. On cliques and path length

The behaviour of the knowledge diffusion system we consider changes as the amount of randomness in network architecture changes. Structural properties have been encapsulated in two statistics: average path length, and cliquishness. The effect of average path lengths is relatively obvious intuitively—a short path increases the diffusion power of the system; knowledge will move to different parts of the graph more quickly, and will degrade less. What about cliquishness? A cliquish graph has the property of local redundancy in the sense that if agents i and j are in the same clique, there are many short paths by which knowledge can pass between them. This is central in this model due to the imperative of double coincidences of wants. In general in an agent's neighbourhood, there will be knowledge held by neighbours which would be of value to the agent but which cannot be directly transmitted. But the more cliquish is the neighbourhood, the more likely it is that there is a short path, through a common neighbour, over which this information can be transmitted. So in general an agent will receive more knowledge from his neighbours. This is where the value of cliquishness arises.

To test this, note that an implication of this difference between cliquish and non-cliquish structures is that the process will stop for different reasons. Recall that there are two conditions that stop trading: either the agents' knowledge levels are too similar (Eq. (4)); or there is a failure of double coincidence of wants (Eq. (2)). If the argument above is correct, then in a cliquish graph, because knowledge moves easily over indirect routes within a neighbourhood, agents become more similar to their neighbours. This will not happen in a non-cliquish graph. Thus in a cliquish world the process is more likely to be stopped by agents being too similar, whereas in a non-cliquish world, termination of the process by a failed double coincidence of wants would be relatively more common. This is seen in Fig. 8, where we show the share of linked-agent pairs in which one agent weakly dominates the other at the end of the process.

Because density is held constant in the family of networks we consider, there is necessarily a tension between cliquishness and path length. In order to decrease path length, cliquishness must also decrease. Thus we expect to find, and do find, an interior maximum. And this interior solution occurs in the small world region where path length approximates that of a random network, but cliquishness remains close to that of a highly regular local interaction network.

Other models generate the value from cliquishness from different mechanisms. But in both of the cases we discuss below, cliquishness is a good thing for reinforcing behaviour similarly to our model in which cliquishness promotes similarity in higher knowledge levels among nearby agents.

Young (1998, Chapter 6) considers the issue of seeding a new behaviour B in a population initially coordinated on the same action A of a symmetric 2×2 Coordination

¹² The re-wiring procedure of Watts and Strogatz, which we employ, is in fact a special case of the procedure presented by Hedström (1994).

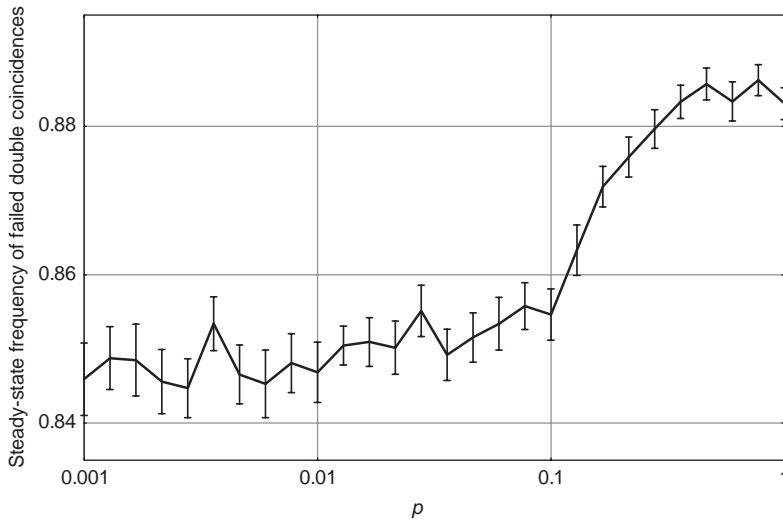


Fig. 8. Average frequency of failed double coincidences of wants in the steady state as a function of p .

Game. To illustrate, assume a pure coordination game where coordinating on A yields 2 and B yields 3. Define a set of agents S to be r -cohesive ($r \in [0, 1]$) if any $i \in S$ has at least a fraction r of his interactions in S , i.e. $\#(S \cap \Gamma_i) / \#\Gamma_i \geq r$ (Morris, 2000). Assume such a group can be found in the population and converted to B . The group is stable if $r > r^* = 2/(2 + 3)$. Action B has a chance to invade the population under the best reply process if and only if there exists a sequencing of individuals in $I - S$, say i_1, i_2, \dots, i_z such that $S \cup \{i_1, \dots, i_j\}$ is at least $2/5$ -cohesive for $i_1 \leq j \leq i_z$. Obviously in a random graph it is usually not the case that such a sequencing exists—it is even hard to find a cohesive starting group S . Here there is value to cohesiveness because an individual only adopts some behaviour when enough neighbours do so. If successive cohesive subsets of individuals cannot be found, contagion stops. In this case however there is no value to path length, as there is a pure epidemic effect, for which nothing degrades over transmissions.

Important insights on the value of cliques are also found in the analysis by Chwe (1999, 2000). Again the issue is to seed behaviour B when everyone initially plays A in a coordination game. Switching from A to B takes place if there are at least m other players who do so in my neighbourhood. I know everybody in my neighbourhood also demands m switches and I know whether my neighbours know each other. So it is fairly straightforward to see that a person switches to B if his neighbourhood contains a clique of size m (that is a complete subset of m players). Cliques are a good thing again. Here it is because, as Chwe (1999, p. 142) puts it, they permit agents to form common knowledge at a local level; if a friend of one of i 's friends is also a friend of i , then common knowledge among the three of them is quickly formed. If m is low this is exactly what is needed. If m is high, common knowledge must be formed among large groups of people; then weak links (that is non cliquish networks) are usually

better because they speed up communication. Granovetter (1973) is mostly concerned with that second effect: a society with weak links, that is to say low neighbourhood cohesiveness or cliquishness but rather a random structure, is better connected in the sense that is rapidly traversed: everyone is within a small number of steps. Weak ties (in the context of our model, the ‘acquaintance ties’ could be interpreted as those not contributing to cliquishness) transmit information throughout the system, and overcome the shortcomings of strong links in this regard. Weak ties facilitate global diffusion, and strong ties permit small groups of agents to exploit fully (through local diffusion) their aggregate knowledge. These two effects are present in our model.

4. Two networks

In their discussion, Watts and Strogatz conjecture that the small world network structure is ubiquitous. They present data showing that the networks of film co-stars, the power grid in the western US and the neural network of the worm *Caenorhabditis elegans* all have the small world structural features of high cliquishness and short average path lengths. In this section we calculate average path lengths and cliquishness for two knowledge networks: the network of research institutes involved in one activity of the EU’s TSER programme;¹³ and a network of innovating firms participating in the BRITE/EURAM programme.¹⁴ The nature of interactions among agents in the networks differ from each other. In the TSER programme, consortia of research institutes form to perform research contracted to the European Commission. Here, knowledge flows through the contacts of individuals as they share the results of the work they are (more or less independently) performing under these contracts. In the BRITE/EURAM programme knowledge flows between firms when they create and carry through cooperative R&D projects.

In both cases, we count the number of nodes in the network, and the average number of edges or connections. From this we calculate baseline values, using the fact that for the family of graphs that we consider here, one has $C(0) = 3/4(n - 2)/(n - 1)$ and for N large enough, $L(0) \simeq N/(2n)$, $L(1) \simeq \ln N/\ln n$ and $C(1) \simeq n/N$. We then calculate cliquishness for the network ($C_{\text{obs.}}$), and average shortest paths ($L_{\text{obs.}}$). These numbers are presented in Table 2. Normalized cliquishness and path length are presented in

¹³ The Targeted Socio-Economic Research programme is part of the Fourth RTD Framework Programme on the Evaluation of Science and Technology Policy Option in Europe. TSER is one of the main sources for funding in the social sciences at the European level. TSER data are available at <http://www.cordis.lu/tser/src/area1.htm>.

¹⁴ The BRITE/EURAM programme was designed to promote R&D networks within European countries, having the ultimate goal of improving the competitiveness of European manufacturing. The dataset covers many technologies and a heterogeneous set of actors: large and small firms; universities; and other research centres. The data consist of all BRITE/EURAM contracts signed in 1990. Gambardella and Garcia-Fontes (1996) used these data to examine the composition of the research networks (that is, the types of partners and their locations typically involved) that were formed under this programme.

Table 2
The two networks and their structural characteristics

Network	Order	Average degree	Cliquishness			Path length		
			$C_{\text{obs.}}$	$C(0)$	$C(1)$	$L_{\text{obs.}}$	$L(0)$	$L(1)$
TSER prog.	183	8.01	0.830[1.291]	0.643	0.044	3.90[0.342]	11.42	2.50
BRITE/EURAM	438	8	0.831[1.292]	0.643	0.023	4.00[0.146]	27.37	2.92

parentheses. Comparing Table 2 with Fig. 2, we see that both networks have small world structural properties—they are highly cliquish and have short paths. In both cases, the network structure has evolved, whether by design or through self-organization, to a structure that is relatively efficient in diffusing knowledge in the long run. The data sets correspond to the model in different ways, with the TSER data being mostly tightly mapped to it in terms of the knowledge transmission mechanism. The variety in the data, though, permits a more robust conclusion that small world networks are a common means of structuring knowledge transmission.

5. Discussion and conclusions

In this paper we have modelled one of the processes by which knowledge is diffused. As more and more policy makers come to be concerned with the so-called knowledge economy, this diffusion becomes a central policy issue. We have shown that the extent of diffusion is clearly affected by the structure of the network over which the diffusion takes place, and that there is an identifiable region of the space of structures in which diffusion is much more complete than elsewhere. This small world region exists where the proportion of links between an agent and other agents not in his neighbourhood is between 1 and 10 percent of all direct links between agent pairs. Technology policy, especially in Europe, is currently going through a phase (one might be tempted to say craze) in which clustering and localization is seen as an extremely important phenomenon. Policy makers are very keen to find policies which will encourage clustering, and create new ‘Silicon Valleys’ in new places and different industries. This craze is based on the belief that the knowledge transmission mechanisms modelled here are extremely important in any innovation system. Our results indicate, though, that this policy goal must be treated with care; it is possible to have too much clustering. It is very important to maintain or even build strong links outside the cluster.

Our results also raise a problem for any policy maker involved in regional technology or knowledge policy. This is the very old efficiency versus equity trade-off. When network structures result in a high average knowledge level, they also generate a high heterogeneity among agents. That is, the distribution of knowledge levels is relatively unequal. To the extent that distribution remains a policy concern, if knowledge is considered a key input to wealth or income, policies aimed at inducing efficient knowledge diffusion will have to address the consequent distribution of income. But whether or not this concern is real depends on the measure used—if variance is the appropriate

measure of distribution, there is a real problem; if the coefficient of variation is used, there is not.

The model could be extended in several obvious ways. We have taken the network structure as given, and have examined its effect on the knowledge diffusion process. But networks clearly evolve in response to agents' experiences. Thus an agent who has had a very successful exchange with one particular agent is likely to try to return to that agent in the future. By this sort of mechanism the strengths of links between agents will change as they gain experience with the network, and the network structure should emerge, and possibly evolve. Former work on network creation has focused on stable network structures. The most common stable network structures found in this literature are the star, empty and complete networks. Little attention has been paid in that literature to the issue of small worlds, possibly due to the difficulty in characterizing them.

The long-run performance of this economy could be improved if reputations of agents could be taken into account. If an agent can generate a reputation as the provider of useful information he is likely to be able to acquire information on credit so to speak. Others will be willing to give him information based on the belief that in the future, he will provide something useful in return. This will partially mitigate the main problem in barter economies, namely the double coincidence of wants constraint.

In the model in this paper, there is no innovation, only diffusion, which we can interpret as the consequence of a single innovative episode. In general, though, innovations happen continually. Including innovation introduces a potentially complex feedback. It is widely believed that within a cluster ideas meet and interact, producing an environment conducive to rapid creation of new ideas. Thus introducing ongoing innovation introduces another trade-off—in a very cliquish graph, the sort of local interaction conducive to idea creation will be significant. But our results show that very highly cliquish networks have poor diffusion properties. There is, therefore, a new source of the trade-off between production of innovations (which would be favoured by a cliquish world) and the diffusion of those ideas (facilitated by short path lengths), both of which are 'good things' for wealth creation.

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